Sunday Times Teaser 3195 – Garden Divide

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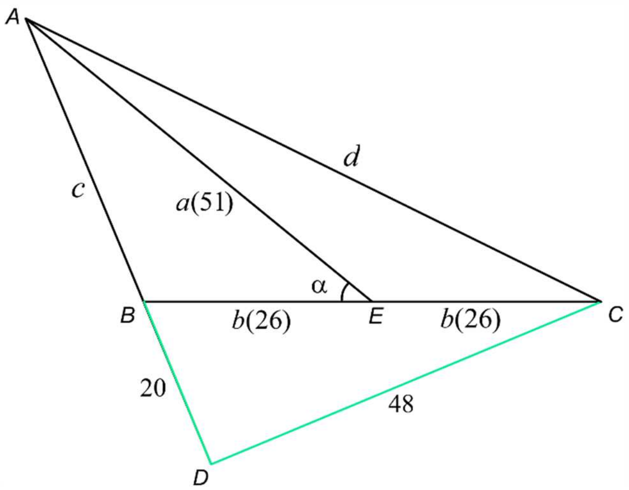
I have a triangular-shaped garden, the sides of which are an exact number of feet long. To improve its usefulness, I've decided to partition it by building a straight fence from one corner to the centre of the opposite side. The length of this fence is exactly 51 feet and the side it attaches to is now 26 feet long each side of the fence.

What, in ascending order, are the lengths of the other two sides of my garden?

Solution by Truner Opec (documented by Brian Gladman)



The garden is shown on the left as triangle *ABC* which is divided by a 51 feet long fence from *A* to *E*, the midpoint of side *BC* (which is 52 feet long).

Using the cosine rule for triangles *ABE* and *AEC*:

$$c^2 = a^2 + b^2 - 2ab\cos(\alpha)$$

$$d^2 = a^2 + b^2 + 2ab\cos(\alpha)$$

and hence:

$$c^2 + d^2 = 2(a^2 + b^2)$$

Noticing that the length of side *BC* is 4 times 13 brings to mind a 5-12-13 Pythagorean triangle which suggests the construction of a "scaled by four" version of such a triangle on the side *BC* as shown in green above. This also creates a second Pythagorean triangle *ADC* for which:

$$(c + 20)^2 + 48^2 = d^2$$

This shows that we are looking for such a triangle with $c < 51 < d$. Using a table of Pythagorean triples indexed by side length (as available on this site), we can find only three possibilities:

c* + *20	***c***	***d***	***a***
36	16	60	~35.38
55	35	73	51
64	44	80	~59.10

Using the equation derived earlier allows a to be calculated using:

$$a = \sqrt{\frac{c^2 + d^2 - 2b^2}{2}}$$

giving the results shown in the fourth column above. Hence there is a solution with $a = 51$ which gives the lengths of the other two sides of the garden as 35 and 73 feet.