

Sunday Times Teaser 3193 - Balanced Education

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George and Martha are headmaster and secretary of Millimix School. There are 1000 pupils divided into 37 classes of at least 27 pupils each; each class has at least 13 members of each sex. Thus with $27 \times 37 = 999$, one class has an extra pupil. The classes are numbered 1-37.

Martha noted that, taking the class with the extra pupil, and adding its class number to the number of girls in that class and a power of two, she would arrive at the square of the class number. Furthermore, the class number equalled the number of classes in which the girls outnumbered the boys.

How many boys are in the school?

Solution by Geoff Rounce.

The teaser requirements can be expressed as:

$$c + g + 2^x = c^2 \Rightarrow c(c - 1) = g + 2^x \quad (1)$$

where g is the number of girls in the class c with an extra pupil and x is a positive integer.

Looking at the equation, it can be seen that:

- 1) $c(c - 1)$ is even because it is the product of consecutive integers.
- 2) 2^x is even because x is a positive integer.
- 3) Hence the third term in the equation, g , must also be even.
- 4) g must be one of (13, 14 or 15), leading to $g = 14$

From equation (1):

LHS of equation:

c	1	2	3	4	5	6
$c - 1$	0	1	2	3	4	5
$c(c - 1)$	0	2	6	12	20	30

RHS of equation:

g	14	14	14	14	14	14
x	0	1	2	3	4	5
$g + 2^x$	15	16	18	22	30	46

By inspection, LHS = RHS when $c = 6$ and $x = 4$.

The total number of girls is hence:

$$g + 14c + 13(36 - c) = g + c + 468$$

which is 488, making the number of boys equal to $1000 - 488$, which is 512.

The answer is 512 boys.