Blindfold Roulette

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Our Roulette wheel has fewer than 100 equal-sized sectors. Before each game a fixed number (fewer than half) of the sectors are randomly designated winning ones and the others as losing. A player can see which are the winning sectors, then is blindfolded. A ball is then placed at random in a winning sector and the player chooses to spin (S) the wheel or move (M) the ball clockwise by one sector. The game continues from there (the player has the same choices) and ends when the ball lands in a losing sector.

Alf's longest winning run was MMSM while Bert's was SSS and Charlie's was MMMS. Being expert logicians, they had always chosen the option that gave the better chance of the ball landing in a winning sector. Remarkably, the probabilities of achieving each run of success were the same.

How many sectors are there and how many are winning sectors?

Solution

Let the number of sectors be N, of which W are winning. Spinning the wheel (S) gives a chance of winning as W/N. The chance of winning by moving (M) depends on the distribution of the winning sectors. For example, if there are ten winning sectors in groups 4 2 2 2, then each group has one sector at the clockwise end which will lose after M, so the chance of winning is 6/10. This equals R1/W, where R1 is the remaining total after removing 1 from each of the groups.

If you choose M and win, this restricts the possible sectors your ball is in, so the odds of winning with M will change next time, while S is fixed. Call the first move M1, the second M2 and so on. In the above example, again remove 1 from each of the remaining possible groups (3 1 1 1), to obtain R2 and M2=R2/R1. Remove 1 from the remaining possible group (2), to obtain R3 and M3=R3/R2. Hence, R1=6, R2=2, R3=1, R4=0 and M1 = 6/10, M2 = 2/6, M3 = 1/2 and M4 = 0. The longest possible run with M is MMM, with probability (6/10)*(2/6)*(1/2) = 1/10, which equals R3/W, after which M is guaranteed to lose.

<u>Charlie MMMS</u>
Now M1*M2*M3 = R3/W, S=W/N, and M1*M2*M3*S=S*S*S (same probability for Charlie and Bert).
Then R3/W = (W/N)*(W/N) or R3*N*N = W*W*W and N<100
Solutions for R3=1 are: (W,N) = (4,8), (9,27) and (16,64), with S=1/2, 1/3 and 1/4, but fewer than half of the sectors are winning, which rules out S=1/2. For larger values of R3, simply multiply W and N by R3. The only possibilities are:
(R3,W,N) = (1,9,27), (1,16,64), (2,18,54) and (3,27,81)

<u>Alf MMSM</u>
We are told that MMSM and SSS give the same odds, so M1*M2*S*M1 = S*S*S. We must have M1>S and M2>S (for Alf to get the better chance with M), but M3<S (for Alf to get the better chance with S).
M1 = R1/W, M2 = R2/R1, S=W/N, so R1*R2 = W*W*W*W/(N*N)

Taking the above possibilities in turn:
W=9, N=27, R1*R2=9, which is impossible with R2<R1<W
W=16, N=64, R1*R2=16, giving R2=2 and R1=8, but M2 (2/8) is not greater than S (1/4)
W=18, N=54, R1*R2=36, giving R2=3 and R1=12, but M2 (3/12) is not greater than S (1/3)
W=18, N=54, R1*R2=36, giving R2=4 and R1=9, which works, giving a possible solution as below
W=27, N=81, R1*R2=81, which is impossible with R2<R1<W

<u>Possible winning sector groups for Alf</u>

3 3 3 3 2 1 1 1 1	W=18	M1=9/18 > 1/3
2 2 2 2 1	R1=9	M2=4/9 > 1/3
1 1 1 1	R2=4	M3=0

M1*M2*S*M1 = SSS = 1/27

<u>Possible winning sector groups for Charlie</u>

4 4 2 2 2 2 2	W=18	M1=11/18 > 1/3
3 3 1 1 1 1 1	R1=11	M2=4/11 > 1/3
2 2	R2=4	M3=2/4 > 1/3
1 1	R3=2	M4=0

M1*M2*M3*S = SSS = 1/27

For Bert, the winning sectors could all be singles, so M1=0 and S would always be the better option.

Answer: 54 and 18