Sunday Times Teaser 3172 – Light Show

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My bedside clock displays the time and date using eight digits; for example at 9.43am on 28th June the display would be (see above).

Each digit in the electronic display lights up some (or all) of seven light segments, the above display lighting up a total of 45 segments.

On one occasion recently the displayed digits were all different and the total number of lit segments was prime. The same was true exactly one day later. Then, just one minute after the second occasion, the number of lit segments was the average of the numbers of lit segments on those two previous occasions.

What was that third display?

Solution by Ciaran Lewis

We can quickly establish some boundary conditions and constraints.

- Using all 10 digits (0-9) requires a total of 49 lit segments. Hence, with only 8 different digits the number of lit segments, S8, is in the range 36 to 44.
- First and second clock readings (R1 and R2) are prime and must be from S8 = 37, 41 or 43.
- Labelling the clock reading format as [***ab.cd ef.gh***]: ***a***, ***e*** and ***g*** are one of (0,1,2), (0,1,2,3) and (0,1) respectively; but ***e*** cannot be 3 since adding a day to ***ef*** = 30 or ***ef*** = 31 leads to the transitions 30 → 31 or 31 → 01 each of which use both 0 and 1 leaving no digit for ***g***; we hence see that (***a, e, g***) are (0, 1, 2) in some order. As ***g*** =1 forces ***h*** to be 0 or 2 we need ***g*** =0.
- If S8 = 43, the two unused digits must contribute 6 segments and can hence only be 1 and 4. Since 1 must be used in R1, S8 = 43 is not a viable option.
- If S8 = 41, the two unused digits must contribute 8 segments and the only options are 3 or 5 with 7 (since 0, 1 and 2 must be used in R1).
- If S8 = 37, the two unused digits must contribute 12 segments and the only options are 3 or 5 with 8 or 6 with 9.

If we assume S8 does not change between R1 and R2, then the change ΔS8=0 can only result from a change of ***f*** = 2 to 3 but, since 2 (i.e. ***a*** or ***e***) is already in use, this is not possible.

Hence, R1 to R 2 must be described by S8 = 41 to 37 or 37 to 41.

- But ΔS8 = −4 can only result from a change of ***f*** = 0 to 1 and since 1 (i.e. ***a*** or ***e***) is already in use this is not possible.
- However, ΔS8 = +4 is possible for a day change ***f*** =7 to 8 and this is the basis of a solution with S8=37 for R1 and S8=41 for R2.
- Since we need S8=37 for R1, then we see that digit 8 is unused along with either 3 or 5.

Since this is the only valid scenario for R1 to R2, we now need to find a transition from R2 to R3 which offers a change ΔS8=−2 to allow S8 for R3 to be the average for R1 and R2.

This is uniquely allowed with the following clock readings using the minute change 9 to 0:

R1 16.59 27.04 (S8 = 37, unused digits 3 and 8)

R2 16.59 28.04 (S8 = 41, unused digits 3 and 7)

R3 17.00 28.04 (S8 = 39) — the answer to the teaser.

Note that ***ae*** = 12 because ***a*** cannot equal 2 (since the hours 20, 21 22 and 23 are all forbidden).