Sunday Times Teaser 3151 – Plant Stock

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A garden centre bought 500 plants of four different varieties from its supplier. The price per plant of each variety was a whole number of pence and their total average price worked out at exactly one pound.

The number of plants of variety 2 purchased was "d" greater than that of variety 1, and its price per plant was "d" pence less than that of variety 1. Similarly, the number of variety 3 plants equalled the number of variety 2 plus "d" and its price equalled the variety 2 price less "d" pence. Finally, the number of variety 4 plants equalled the number of variety 3 plus "d" and its price equalled the variety 3 price less "d" pence.

What, in pence, is the most that a plant could have cost?

Solution by Brian Gladman

Let n be the number of plant variety 1 that is bought at cost p per plant. The numbers and cost for the four varieties are hence:

$$\begin{array}{llll} n & n+d & n+2d & n+3d \\ p & p-d & p-2d & p-3d \end{array}$$

The total numbers and total cost are hence:

$$4n + 6d = 500 \quad (1)$$

$$4np - 6nd + 6pd - 14d^2 = 50000 \quad (2)$$

which can be simplified and rearranged as:

$$\begin{aligned} n &= (250 - 3d)/2 \\ p &= \{(d + 75)^2 + 4375\}/100 \end{aligned} \quad (3)$$

Note that p is a monotonically increasing function of d, which means that p_{max} occurs when d_{max} occurs. Since $n \geq 0$, equation (1) shows that d_{max} is at most 83, so we need the largest d value that is 83 or less which gives an integer value of p in equation (3).

A few trials give this as $d = 80$, showing that the most expensive plant could cost at most £2.84, of which 5 could be bought.