Curling League

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In our curling league (more than 4 teams but fewer than 26), each team plays each other once. Teams are ranked according to the number of wins (draws are impossible). If any teams are tied on wins, ranking is only possible if those teams have different numbers of wins in their mutual games. For example, in a three-way tie if A beats B, B beats C and A beats C, the ranking is ABC, but if C beats A (or A has not yet played C), then ranking is impossible, as A and B have one win each.

At one point (each team had played G games), ranking the teams as above was possible. However, if G was any smaller, a ranking would have been impossible, irrespective of results. With one more team in the league, the minimum number of games needed to allow a ranking is G+2.

How many teams are in the league and what was the value of G?

Solution **Answer: 16 and 6**

Suppose there are T teams, each having played G games.

If two teams are tied, they can be ranked if their mutual game has been played. For three tied teams, they can each have played at most two mutual games. In order for them to be ranked, the mutual wins must be 2, 1 and 0, so all the three mutual games must have been played. Similarly, for four tied teams to be ranked with 3, 2, 1 and 0 wins, all the six mutual games must have been played.

More than one team with G wins is impossible because all the mutual games have been played in order to rank the teams, so at least one game has been lost by one of the tied teams. Similarly, there can be no more than two teams with G-1 wins, no more than three teams with G-2 wins and so on. Working up from the other end of the table, more than one team with 0 wins is impossible, because the mutual games have been played. Similarly, there can be no more than two teams with 1 win, no more than three teams with 2 wins and so on. For instance, if G=3, the maximum number of wins comprises 1 team with 3 wins + 2 teams with 2 wins + 2 teams with 1 win + 1 team with 0 wins, i.e. 9 wins. For G=4, you can have (1x4)+(2x3)+(3x2)+(2x1)+(1x0) = 18. The full list is:

G	2	3	4	5	6	7	8	9
Max number of wins	4	9	18	30	48	70	100	135

[For an even value of G, the max number of wins is G(G+2)(G+2)/8. For odd G it is G(G+1)(G+3)/8]

This has to be at least the number of games played by the T teams, which is TG/2, so for a given value of T you can easily work out the minimum value of G (remembering that if T is odd, it is impossible for all of the teams to have played an odd number of games, so G is even).

T	5	6	7	8	9	10	11	12	13	14	15
Min value of G	4	3	4	4	4	5	6	5	6	6	6

T	16	17	18	19	20	21	22	23	24	25	26
Min value of G	6	8	7	8	7	8	8	8	8	8	9

We are told that for the addition of one team, the minimum value of G increases by 2, so T must be 16 and G is 6. The numbers of wins for the teams are 6 5 5 4 4 4 3 3 3 3 2 2 2 1 1 0, with 48 games played at that point.

A possible table of results (with tie-breaking mutual results underlined) is:

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	wins
A		W		W							W			W	W	W	6
B	L		W		W			W					W	W			5
C		L		W			W	W	W					W			5
D	L		L		W	W						W				W	4
E		L		L		W					W		W		W		4
F				L	L						W	W			W	W	4
G			L					W	W	W		L	L				3
H		L	L				L		W	W						W	3
I			L				L	L		W				W	W		3
J							L	L	L		W			W		W	3
K	L				L	L				L		W	W				2
L				L		L	W				L		W		L		2
M		L			L		W				L	L				W	2
N	L	L	L						L	L					W		1
O	L				L	L			L			W		L			1
P	L			L		L		L		L			L				0