Sunday Times Teaser 3140 - Enjoy Every Minute

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On rugby international morning, I found myself, along with eight friends, in a pub 5.8 miles from the match ground. We were enjoying ourselves, and so wished to delay our departure for the ground until the last possible minute. The publican, wishing to keep our custom for as long as possible, offered to help us get there by carrying us, one at a time, as pillion passengers on his motorbike.

We could walk at 2.5mph and the bike would travel at 30mph. We all left the pub together, and arrived at the ground in time for kick-off.

Ignoring the time taken getting on and off the bike, what was our minimum travelling time in minutes?

Solution by Brian Gladman (see also Jim Randell at https://s2t2.home.blog/2022/11/25/teaser-3140-enjoy-every-minute/#comment-4389).

In order to minimise the total time taken for the friends to travel from the pub to the playing ground, it is important that the barman and all the friends start moving at the same time and then move continuously at their respective speeds until they all arrive simultaneously at the playing ground.

Let the distance from the pub to the ground be d, the pedestrian walking speed of the n friends be p and the speed of the barman on the motorbike be b. If the friends each walk a distance w, the time taken to complete their journey will be:

$$t = \frac{w}{p} + \frac{d - w}{b} \tag{1}$$

The barman will travel backwards and forwards a total of $2n - 1$ times and will travel $2n - 1$ times the distance d less twice the total distance walked by the friends. The latter is doubled because, in picking up and dropping off friends before the two end points, the barman will save twice this distance since he would have had to travel it twice. The barman will hence take a total time given by:

$$t = \frac{(2n - 1)d - 2nw}{b} \tag{2}$$

Equating these two times and solving for w now gives:

$$w = \frac{2(n - 1)p}{(2n - 1)p + b} d = \frac{(k - 1)p}{kp + b} d \tag{3}$$

where $k = 2n - 1$. We can substitute for w in equation (1) and simplify the result to find the minimum time for the transfer:

$$t_{min} = \left(\frac{kb + p}{kp + b}\right)\left(\frac{d}{b}\right) \tag{4}$$

Substituting the values into this equation now gives the total transfer time in minutes as:

$$t_{min} = 60\left(\frac{17 \times 30 + 2.5}{17 \times 2.5 + 30}\right)\left(\frac{5.8}{30}\right) = \left(\frac{205}{29}\right)\left(\frac{58}{5}\right) = 82 \text{ minutes} \tag{5}$$