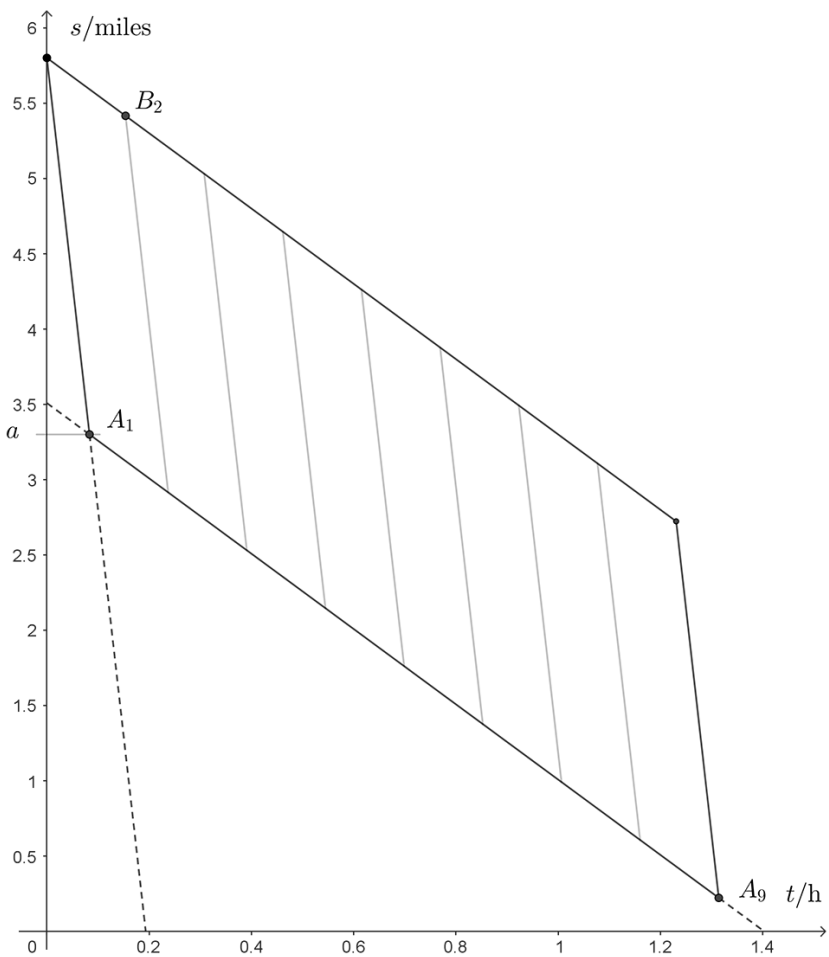
Sunday Times Teaser 3140 by Howard Williams

Enjoy Every Minute

On rugby international morning, I found myself, along with eight friends, in a pub 5.8 miles from the match ground. We were enjoying ourselves, and so wished to delay our departure for the ground until the last possible minute. The publican, wishing to keep our custom for as long as possible, offered to help us get there by carrying us, one at a time, as pillion passengers on his motorbike.

We could walk at 2.5mph and the bike would travel at 30mph. We all left the pub together, and arrived at the ground in time for kick-off.

Ignoring the time taken getting on and off the bike, what was our minimum travelling time in minutes?



The diagram on the left is a time/distance graph with time (t) running rightwards on the x-axis and distance (s) downwards on the y-axis. The variable a is the distance from the playground where the first guest is dropped off to walk the remaining part.

B_2 is where/when guest no. two is picked up to have his turn with the bike, and A_9 is where/when the motorbike with the last guest catches up with the other guests walking together.
The optimal place for that would be at the match ground.

Coordinates for A_1 and B_2 are

$$A_1: \left(\frac{5.8\ -a}{30}, a\right)$$
$$B_2: \left(\frac{11.6\ -2a}{32.5}, \frac{2a-63.8}{13}\right)$$

From these it follows that

$$A_9: \left(\frac{1189\ -205a}{390}, \frac{29a-92.8}{13}\right)$$

Now a must be chosen so that $(t, s) = (t, 0)$ for A_9. We get

$$\frac{29a-92.8}{13}=0 \quad \Rightarrow \quad \underline{a=3.2}\ \text{(miles)}$$

If the publican manages to drop his first passenger 3.2 miles from the match ground, the rest is easy..

The corresponding t is

$$t=\frac{1189-205\cdot 3.2}{390}\text{h}=\frac{533}{390}\text{ h}=\ \underline{82\text{ minutes}}$$

Erling Torkildsen, November 27 2022