Sunday Times Teaser 3126 – Sweet Success

by Victor Bryant

Published Sunday August 21 2022

My five nieces Abby, Betty, Cathy, Dolly and Emily each had some sweets. I asked them how many they had but they refused to answer directly. Instead, in turn, each possible pair from the five stepped forward and told me the total number of sweets the two of them had. All I remember is that all ten totals were different, that Abby and Betty's total of 8 was the lowest, and that Cathy and Dolly's total of 18 was the second highest. I also remember one of the other totals between those two but I don't remember whose total it was. With that limited information I have worked out the total number of sweets.

In fact, it turns out that the other total I remember was Betty and Cathy's.

In alphabetical order of their names, how many sweets did each girl have?

Solution by Ciaran Lewis

General initial observations:

1. ABCDE values are all different and none is zero (otherwise there would be duplicated pair sums).
2. The ten pair sums will be the same no matter which order values A to E are assigned.

This means we can use A↔B = 8 (e.g. 3+5 ≡ 5+3 = 8) and C↔D = 18. Note that there are 4 possible values of (B+C) for any permissible A↔B, C↔D combination. Also, we need only consider A↔B = 1↔7, 2↔6 or 3↔5.

Dropping the interchangeable symbols (↔) the only 6 options allowed are:

(1) 1, 7, 8, 10, E	(2) 2, 6, 7, 11, E	(3) 2, 6, 8, 10, E
(4) 3, 5, 6, 12, E	(5) 3, 5, 7, 11, E	(6) 3, 5, 8, 12, E

Immediately, we see E must be greater than 6 to allow a pair sum greater than 18; and E must be less than D to avoid two pairs greater than 18 (if $E > D$ then $C + D < C + E < D + E$). Hence E is less than 12.

Considering E = 7

A↔B = 2↔6 or 3↔5 (ie can't repeat 7).

For A↔B = 2↔6, C↔D = 8↔10 is the only option (ie can't have 7 or new pair sums ≤ 8).

But 2, 6, 8, 10, 7 sweets does not allow a pair sum ≥ 18.

For A↔B = 3↔5, C↔D = 6↔12 is the only option.

Here, the pair sums are 8, 9, 10, 11, 12, 13, 15, 17, 18, 19 and (B+C) equals 9, 11, 15 or 17.

Considering E = 8

A↔B = 1↔7, 2↔6 or 3↔5

For A↔B = 1↔7 or 2↔6, C↔D has no options

For A↔B = 3↔5, C↔D = 7↔11 is the only option.

Here, the pair sums are 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 21 and (B+C) equals 10, 12, 14 or 16.

Considering E = 9

With A↔B = 2↔6, C↔D = 8↔10 is the only option.

Here, the pair sums are 8, 10, 11, 12, 14, 15, 16, 17, 18, 19 and (B+C) equals 10, 12, 14 or 16.

Considering E = 10

With A↔B = 3↔5, C↔D = 7↔11 is the only option.

Here, the pair sums are 8, 10, 12, 13, 14, 15, 16, 17, 18, 21 and (B+C) equals 10, 12, 14 or 16.

Considering E = 11

Options (2) and (5) are excluded and option (4) gives two pair sums equal to 17. Other options give two pair sums >18. Hence, E = 11 is excluded.

We see that for E = 8, 9 or 10, the values for (B+C) are the same and hence can't be used to distinguish the number of sweets for E. We conclude that E = 7 and that (B+C) = 9, 11, 15 or 17.

We further note that (B+C) = 11, 15 and 17 appears in all 16 pair sum sets and can't be used to distinguish between options. We conclude (B+C) = 9 is unique with B = 3 and C = 6.

Finally, we have A, B, C, D, E given by 5, 3, 6, 12, 7.