Sunday Times Teaser 3124 – Lawn Order

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A gardener was laying out the border of a new lawn; he had placed a set of straight lawn edging strips, of lengths 16, 8, 7, 7, 7, 5, 4, 4, 4 & 4 feet, which joined at right angles to form a simple circuit. His neighbour called over the fence, "Nice day for a bit of garden work, eh? Is that really the shape you've decided on? If you took that one joined to its two neighbours, and turned them together through 180°, you could have a different shape. Same with that one over there, or this one over here — oh, look, or that other one." The gardener wished that one of his neighbours would turn through 180°.

What is the area of the planned lawn, in square feet?

Solution by John Crabtree.

The sum of the lengths is 66 feet with all strips joined to each other at right angles, which means that 5 pieces run east / west, and five north / south. The two east / west sides and the two north / south pairs have to have strips whose combined lengths are the same.

Considering the largest strip first:

- if 16 = 4 + 4 + 4 + 4, then 17 + 17 cannot be made for the other two sides;
- likewise, if 16 + 4 = 8 + 7 + 5, then 13 + 13 cannot be made;
- If 16 + 7 = 23, then 23 cannot be made with three remaining pieces.

This leaves 16 + 5 = 7 + 7 + 7 (east / west); and 8 + 4 = 4 + 4 + 4 (north / south).

Three pieces joined together can form either a "C" shape or an "S" shape. A symmetrical "S", when rotated, will not change the shape of the garden

A square has 4 internal right angles (IRAs). The garden has 10 joints and so must have 7 IRAs and 3 external right angles (ERAs). To create a "C" shape that can be rotated without a conflict requires using 2 ERAs, i.e. ERA - IRA - IRA - ERA. or IRA - ERA- ERA - IRA. You cannot have a rectangle with one of the sides incomplete.

As there are only three ERAs in total, there is a maximum of two rotatable "C" shapes that can be created. And so, a minimum of two asymmetrical "S" shapes must be rotated. Only two are possible i.e. 16 - 4//8 - 5 and 8 - 5/7 - 4.

We can now try to synthesize a workable shape for the garden.

If 16 - 8 - 5, i.e. (0, 0) (16, 0) (16, 8) (21, 8), then one has to go upwards from the (21, 8). But rotating the 16 - 8 - 5 piece makes it impossible to go upwards from (0, 0).

And so 16 - 4 - 5, i.e. (0, 0) (16, 0) (16, 4) (21, 4) is one asymmetrical "S" piece, and the other is 8 - 7 - 4. They must connect to each other. If the 8 - 7 - 4 piece connects at (21, 5), it must go down. But then it cannot be rotated

And so, the 8 - 7 - 4 piece connects at (0, 0) and must go up. To allow the 16 - 5 - 4 piece to rotate, the points must be at (0, 8) (7, 8) (7, 12). The complete circuit must then be (0, 0) (16, 0) (16, 4) (21, 4) (21, 8) (14, 8) (14, 12) (7, 12) (7, 8) (0, 8) (0, 0).

The "S" shapes 16 - 4 - 5 and 8 - 7 - 4 can be rotated. The "C" shapes 5 - 4 - 7 and 4 - 7 - 4, which each involve two ERAs, can be rotated.

The area of the garden is hence 8 × 21 + 28 - 20 = 176 sq. feet.

Garden Shape and Area (produced by Brian Gladman)

There are eight different garden shapes, the garden that solves the teaser, four that can be derived from it using ‘three strip rotations’ and three others. In the drawings below, the left-hand column shows the ‘solution’ garden four times adjacent to each of the four transformed shapes in the centre column. The right-hand column shows the three remaining garden shapes. The figures show the areas of the gardens (in square feet).