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Top Marks

A teacher is preparing her end of term class test. After the test she will arrive at each pupil's score by giving a fixed number of marks for a correct answer, no marks if a question is not attempted, and deducting a mark for each incorrect answer. The computer program she uses to prepare parents' reports can only accept tests with the number of possible test scores (including negative scores) equal to 100.

She has worked out all possible combinations of the number of questions asked and marks awarded for a correct answer that satisfy this requirement, and has chosen the one that allows the highest possible score for a pupil.

What is that highest possible score?

Fragment:

Let n be the number of questions and a the marks rewarded a correct answer. In the **R**ight/**W**rong mark-table in symbolic terms below, the total scores that cannot be obtained, appears in the yellow cells forming a triangle with base and latitude $(a-1)$. This demonstrates that the number total scores that cannot be obtained is $a\cdot(a-1)/2$.

R* \ *W	**0**	**1**	**2**	**3**	⋯	***a* – 1**	⋯	***n* – 1**	***n***
n	$n\cdot a$	$n\cdot a-1$	$n\cdot a-2$	$n\cdot a-3$	⋯	$n\cdot a-(a-1)$	–	–	–
***n* – 1**	$(n-1)\cdot a$	$(n-1)\cdot a-1$	$(n-1)\cdot a-2$	$(n-1)\cdot a-3$	⋯	$(n-1)\cdot a-(a-1)$	–	–	–
***n* – 2**	$(n-2)\cdot a$	$(n-2)\cdot a-1$	$(n-2)\cdot a-2$	$(n-2)\cdot a-3$	⋯	$(n-2)\cdot a-(a-1)$	–	–	–
***n* – 3**	$(n-3)\cdot a$	$(n-3)\cdot a-1$	$(n-3)\cdot a-2$	$(n-3)\cdot a-3$	⋯	$(n-3)\cdot a-(a-1)$	–	–	–
⋮	⋮	⋮	⋮	⋮		⋮	–	–	–
***n* – (*a* – 2)**	⋮	⋮	⋮	⋮		$(n-(a-2))\cdot a-(a-1)$	–	–	–
⋮	⋮	⋮	⋮	⋮				–	–
1	a	$a-1$	$a-2$	$a-3$				$a-(n-1)$	–
0	0	-1	-2	-3				$-(n-1)$	$-n$

Number of obtainable scores is then: $n\cdot a+n+1-a\cdot(a-1)/2$.

And the equation to be solved is: $2\cdot n\cdot a+2\cdot n+2-a\cdot(a-1)=200$.

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