Sunday Times Teaser 3111 – Don't Miss a Second

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Published Sunday May 08, 2022

I have an analogue wall clock with a second hand and also a separate 24-hour hh:mm:ss digital clock. The wall clock loses a whole number of seconds over a two-digit period of seconds. The digital clock gains at a rate 2½% greater than the wall clock loses. After resetting both clocks to the correct time, I noticed that they both displayed the same but wrong time later in the same week, and one hour earlier than the time of setting.

I can reset one of the clocks at an exact hour so that it will show the correct time when the televised rugby kicks off at 19:15:00 on the 31st.

What is the latest time (hour and date) when I can do this?

Solution by Brian Gladman

Let the analogue clock lose s seconds in a period of p seconds. With t as the time, t_a and t_d as the displayed times on the analogue and digital clocks:

$$t_a = (1 - s/p)t = \left(\frac{p - s}{p}\right)t \tag{1}$$

$$t_d = (1 + 1.025\, s/p)t = \left(\frac{40p + 41s}{40p}\right)t \tag{2}$$

The difference between the displayed times on the two clocks is:

$$t_d - t_a = \left(\frac{81s}{40p}\right)t \tag{3}$$

In a time of d days less one hour the clocks read the same; hence for some integer k:

$$\left(\frac{81s}{40p}\right)(24d - 1) = 12k \tag{4}$$

which after rearrangement becomes:

$$\frac{s}{p} = \frac{160k}{3^3(24d - 1)} \tag{5}$$

Both s and p are integers less than 100 and $k \leq 16$ since $s < p$. For s to be less than 100, the $(24d - 1)$ term must share a common factor with 160, which occurs when $d = 4$ giving:

$$\frac{s}{p} = \left(\frac{32k}{3^3 \times 19}\right) \tag{6}$$

Since $k \leq 16$, p must be a multiple of 19, the multiple being a power of 3; hence $k = 9$ and $s/p = 32/57$.

When the rugby starts, one of the clocks displays the right time which means that either the analogue clock has lost 12 hours or the digital clock has gained 24. This must happen in an integral number of hours (say h) plus 15 minutes.

Considering the analogue clock first:

$$\left(\frac{s}{p}\right)\frac{4h + 1}{4} = 12k \quad \Rightarrow \quad 4h + 1 = 3 \times 57(k/2) \tag{7}$$

where k must be a multiple of 2 for h to be integer. This gives a smallest h value of 128 when $k = 6$.

For the digital clock:

$$\left(\frac{41s}{40p}\right)\frac{4h + 1}{4} = 24k \quad \Rightarrow \quad 4h + 1 = 120 \times 57(k/41) \tag{8}$$

where k has to be a multiple of 41 for h to be integer. This has no solution since the two sides are odd and even respectively.

Hence the analogue clock has to be set 128 hours before 7PM on 31st May, which is 11AM on 26th May.