Many a Slip

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I have written down three 3-figure numbers in decreasing order and added them up to give their 4-figure sum, which is a perfect square: the digit 0 occurred nowhere in my sum.

Now I have attempted to replace digits consistently by letters and I have written the sum as

CUP + AND + LIP = SLIP

However, there's "many a slip twixt cup and lip" and unfortunately one of those thirteen letters is incorrect. If you knew which letter was incorrect then you should be able to work out the three 3-figure numbers.

What are they?

Solution **Answer: 826, 574 and 536**

	C	U	P
	A	N	D
	L	I	P
S	L	I	P

(a)

If the shaded letters are correct then C>A>L and U≠N. So if you could find a solution CU? + AN? + L?? it follows that CN? + AU? + L?? would be another solution. So in these cases you could not work out my three numbers.

	C	U	P
	A	N	D
	L	I	P
S	L	I	P

(b)

If the shaded letters here are correct then we would require wxP + yzD to be 1000. If we found such a solution we'd have x+z=9 and so x≠z and (as before) wzP + yxD would be a different solution. Hence, once again, in these cases you could not work out my numbers.

(c) It follows from (a) and (b) that the incorrect letter is in both the shaded areas and so it is the L in LIP. Therefore the correct sum now becomes

	C	U	P
	A	N	D
	?	I	P
S	L	I	P

So SLIP is a 4-figure square with four different non-zero digits, P is the final digit of a square, P+D = 10 and P≠5. In particular, if S=1 then P=4 or 6. The only possibilities are

1296 1764 1936 (the next being 2916, too large).

So S=1 and P/D are 4/6 in some order. Also U+N=9 and U/N must be 2/7 in some order. That means that SLIP can only be 1936, making S=1, L=9, I=3, P=6, D=4, U/N=2/7, leaving C(>A) as 8 and A=5. Then we must have ?=5 to complete the sum:

	8	$^2/_7$	6
	5	$^7/_2$	4
	5	3	6
1	9	3	6

But for the three numbers to be in decreasing order they are uniquely determined as

826, 574 and 536