Sunday Times Teaser 3103 – Empowered

by Howard Williams

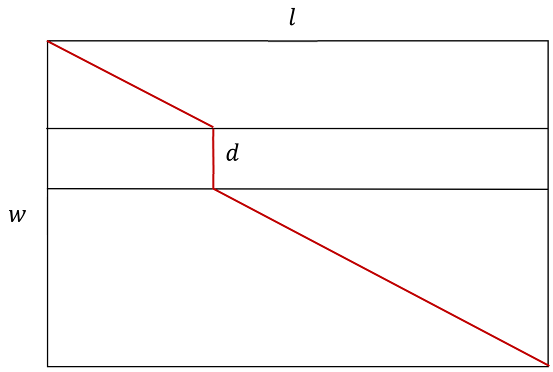
Published Sunday March 13 2022

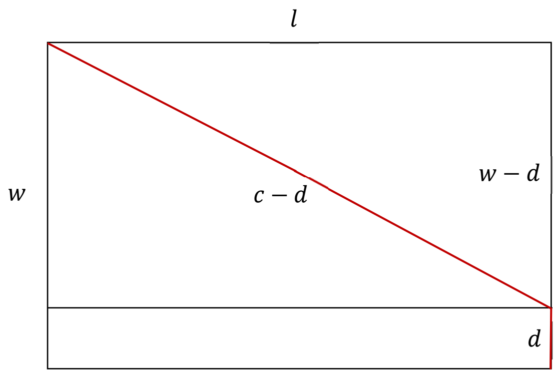
I have a rectangular garden whose sides are whole numbers of feet. Away from the edge, an exposed strip of ground, again a whole number of feet in width, runs straight across (not diagonally) from a shorter side of the garden to the other shorter side. I need to run an electrical cable along the ground, between two opposite corners of the garden. Where the cable crosses the exposed area, it has to be threaded through expensive linear ducting to avoid damage. Because of costs, whilst seeking to minimise the length of cable, my overriding concern is to minimise the length of ducting used.

The straight-line distance between the two corners to be connected is 123 feet, but in minimising costs, the length of cable needed is a whole number of feet longer than this.

What is the length of cable needed?

Solution by Brian Gladman





The drawing to the left shows the garden with the exposed strip and the path taken by the cable across the garden and through the duct. The width and length of the garden are w and l respectively and the duct length is d. Let the length of the garden diagonal be D and the total cable length, including that in the duct, be c.

If we cut the exposed strip out of the garden, move the bottom of the garden up and put the strip at the bottom, we can see from the bottom diagram that the cable length will remain unchanged and this allows us to use this configuration to determine the cable's length.

From the garden's diagonal we have:

$$l^2 + w^2 = D^2$$

with $D = 123$. Tables of Pythagorean triples show only one for this diagonal, giving $w = 27$ and $l = 120$.

For the diagonal $c - d$ we now need a triple with one side of length 120 and a hypotenuse less than 123. Again, there is only one, which gives $w - d = 22$ and $c - d = 122$. Hence $d = w - 22 = 5$ and $c = d + 122 = 127$.

Other approaches are available using the equation:

$$(c - d)^2 = l^2 + (w - d)^2$$

where we know w and l and can increase d in integer steps until the right-hand side is a perfect square giving an integer c. Alternatively, we can solve the equation for d:

$$d = \frac{c^2 - D^2}{2(c - w)}$$

and increase c in integer steps starting at 124 until we obtain an integer value for the duct length d.