Sunday Times Teaser 3090 – Main Line

by Andrew Skidmore

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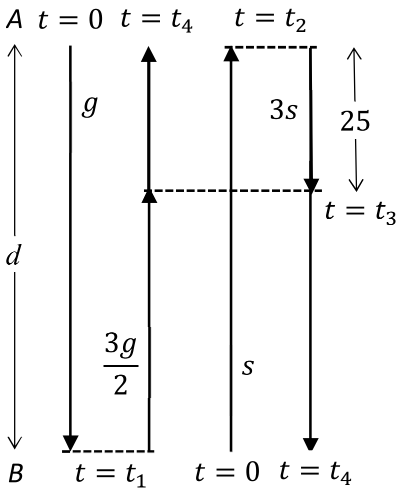
Anton and Boris live next to a railway line. One morning a goods train passed Anton's house travelling south just as a slower train passed Boris's house travelling north. The goods train passed Boris's house at the same time as a passenger train, heading north at a speed that was half as fast again as the goods train. Similarly, as the slower train passed Anton's house it passed a passenger train; this was heading south at a speed that was three times as great as that of the slower train.

The passenger trains then passed each other at a point 25 kilometres from Anton's house before simultaneously passing the two houses.

All four trains travelled along the same route and kept to their own constant speeds.

How far apart do Anton and Boris live?



Solution by Brian Gladman

Let the distance between A and B be d.

Let the goods train travel south at speed g, passing A at time $t = 0$ and reaching B at time t_1.

Let the slower train travel north at speed s, passing B at time $t = 0$ and reaching A at time t_2.

Let the passenger train travel north at speed $3g/2$, passing B at time t_1 and reaching A at time t_4.

Let the final train travel south at speed $3s$, passing A at time t_2 and reaching B at time $t4$.

The latter two trains pass each other 25 kilometres south of A at time t_3.

We can now formulate the following four equations for d:

$$d = gt_1 = st_2 = \left(\frac{3g}{2}\right)(t_4 - t_1) = 3s(t_4 - t_2)$$

giving:

$$t_1 = d/g$$

$$t_2 = d/s$$

$$t_4 = t_1 + \frac{2d}{3g} = \frac{5d}{3g}$$

$$t_4 = t_2 + \frac{d}{3s} = \frac{4d}{3s}$$

Equating the two equations for t_4 now gives the relationship between s and g :

$$5s = 4g$$

From the passing point of the latter two trains, we obtain:

$$t_3 = t_2 + \frac{25}{3s} = \frac{3d + 25}{3s}$$

$$t_3 = t_1 + \frac{2(d - 50)}{3g} = \frac{5(d - 10)}{3g}$$

Equating the two equations for t_3 now gives:

$$(3d + 25)g = 5s(d - 10) = 4g(d - 10)$$

which gives the distance d between A and B as 65 kilometres.