Sunday Times Teaser 3058

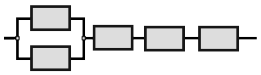
by Peter Good

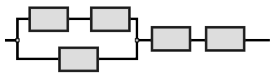
Total Resistance

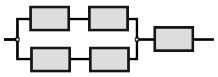
A physics teacher taught the class that resistors connected in serial have a total resistance that is the sum of their resistances while resistors connected in parallel have a total resistance that is the reciprocal of the sum of their reciprocal resistances, as shown in the diagrams. Each pupil was told to take five 35-ohm resistors and combine all five into a network. Each pupil then had to calculate theoretically and check experimentally the resistance of his or her network. Every network had a different resistance and the number of different resistances was the maximum possible. The total sum of these resistances was a whole number.

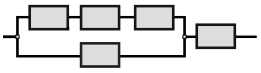
How many pupils were there in the class and what was the sum of the resistances?

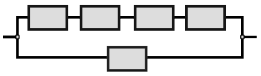
1.  $R_1 = 5r$

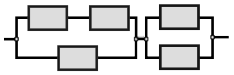
2.  $R_2 = 3r + \frac{r}{2}$

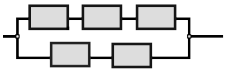
3.  $R_3 = 2r + \left(\frac{1}{r} + \frac{1}{2r}\right)^{-1} = 2r + \frac{2r}{3}$

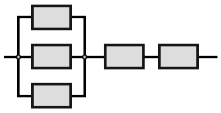
4.  $R_4 = r + \left(\frac{1}{2r} + \frac{1}{2r}\right)^{-1} = 2r$

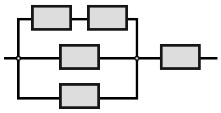
5.  $R_5 = r + \left(\frac{1}{r} + \frac{1}{3r}\right)^{-1} = r + \frac{3r}{4}$

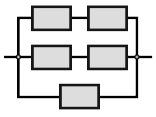
6.  $R_6 = \left(\frac{1}{r} + \frac{1}{4r}\right)^{-1} = \frac{4r}{5}$

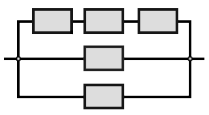
7.  $R_7 = \left(\frac{1}{r} + \frac{1}{2r}\right)^{-1} + \frac{r}{2} = r + \frac{r}{6}$

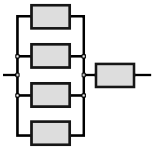
8.  $R_8 = \left(\frac{1}{3r} + \frac{1}{2r}\right)^{-1} = r + \frac{r}{5}$

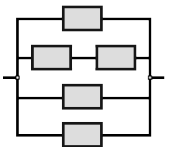
9.  $R_9 = 2r + \frac{r}{3}$

10.  $R_{10} = r + \left(\frac{1}{2r} + \frac{1}{r} + \frac{1}{r}\right)^{-1} = r + \frac{2r}{5}$

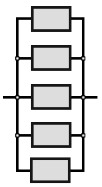
11.  $R_{11} = \left(\frac{1}{2r} + \frac{1}{2r} + \frac{1}{r}\right)^{-1} = \frac{r}{2}$

12.  $R_{12} = \left(\frac{1}{3r} + \frac{1}{r} + \frac{1}{r}\right)^{-1} = \frac{3r}{7}$

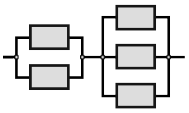
13.  $R_{13} = r + \frac{r}{4}$

14.  $R_{14} = \left(3 \cdot \frac{1}{r} + \frac{1}{2r}\right)^{-1} = \frac{2r}{7}$

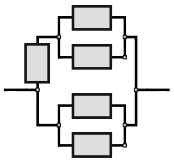
(cont.)

15. 

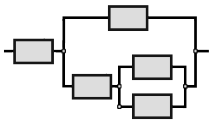
$$R_{15} = \frac{r}{5}$$

16. 

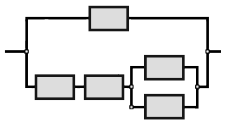
$$R_{16} = \frac{r}{2} + \frac{r}{3} = \frac{5r}{6}$$

17. 

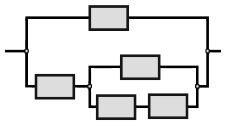
$$R_{17} = \left(\frac{2}{3r} + \frac{2}{r}\right)^{-1} = \frac{3r}{8}$$

18. 

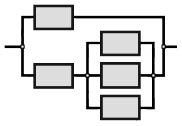
$$R_{18} = r + \left(\frac{1}{r} + \frac{2}{3r}\right)^{-1} = r + \frac{3r}{5}$$

19. 

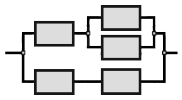
$$R_{19} = \left(\frac{1}{r} + \frac{2}{5r}\right)^{-1} = \frac{5r}{7}$$

20. 

$$R_{20} = \left(\frac{1}{r} + \frac{3}{5r}\right)^{-1} = \frac{5r}{8}$$

21. 

$$R_{21} = \left(\frac{1}{r} + \frac{3}{4r}\right)^{-1} = \frac{4r}{7}$$

22. 

$$R_{22} = \left(\frac{1}{2r} + \frac{2}{3r}\right)^{-1} = \frac{6r}{7}$$

$$R = \sum_{i=1}^{22} R_i = 30r + \frac{2r}{35}$$

$$\underline{r = 35\ \Omega \implies R = 1052\ \Omega}$$

There were 22 pupils in the class and the sum of the resistances was 1052 ohm.

Erling Torkildsen, May 3 2021