Sunday Times Teaser 3078 – Total Resistance

by Peter Good

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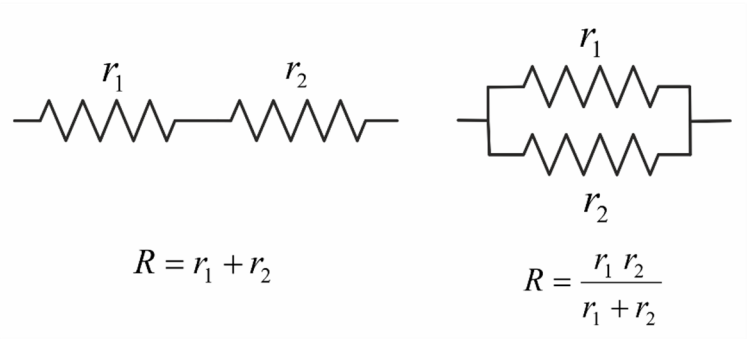
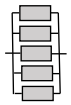
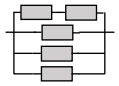
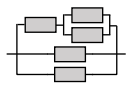
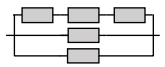
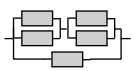
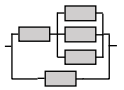
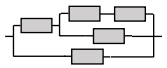
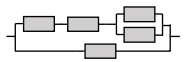
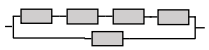
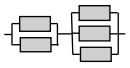
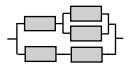
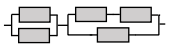
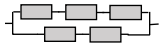
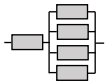
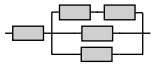
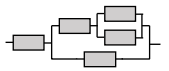
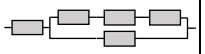
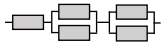
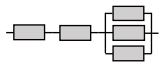
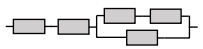
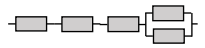
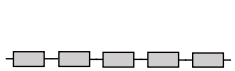
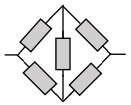


A physics teacher taught the class that resistors connected in serial have a total resistance that is the sum of their resistances while resistors connected in parallel have a total resistance that is the reciprocal of the sum of their reciprocal resistances, as shown in the diagrams. Each pupil was told to take five 35-ohm resistors and combine all five into a network. Each pupil then had to calculate theoretically and check experimentally the resistance of his or her network. Every network had a different resistance and the number of different resistances was the maximum possible. The total sum of these resistances was a whole number.

How many pupils were there in the class and what was the sum of the resistances?

Solution by Brian Gladman

There are 23 arrangements of the five resistors as shown below. The values to the right of each arrangement give the resistance when the networks are constructed using 1-ohm resistors. The final network, called a bridge, cannot be constructed using serial and parallel configurations and is not intended to be a part of the solution. The sum excluding this arrangement is $1052/35$ ohms, which gives a sum of 1052 ohms when the networks are constructed using 35-ohm resistors. If the bridge were to be included the sum would be 1087 ohms, which offers a neat alternative solution since it is a prime.

	$\frac{1}{5}$		$\frac{2}{7}$		$\frac{3}{8}$		$\frac{3}{7}$		$\frac{1}{2}$		$\frac{4}{7}$			
	$\frac{5}{8}$		$\frac{5}{7}$		$\frac{4}{5}$		$\frac{5}{6}$		$\frac{6}{7}$					
	$\frac{7}{6}$		$\frac{6}{5}$		$\frac{5}{4}$		$\frac{7}{5}$		$\frac{8}{5}$					
	$\frac{7}{4}$		2		$\frac{7}{3}$		$\frac{8}{3}$		$\frac{7}{2}$					
	5		1											