Discs a Go-Go

My kitchen floor is tiled with identically-sized equilateral triangle tiles while the floor of the bathroom is tiled with identically-sized regular hexagon tiles, the tiles being less than 1m across. In both cases the gaps between tiles are negligible. After much experimenting I found that a circular disc dropped at random onto either the kitchen or bathroom floor had exactly the same (non-zero) chance of landing on just one tile.

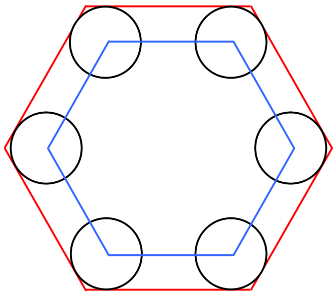
The length of each side of the triangular tiles and the length of each side of the hexagon tiles are both even triangular numbers of mm (ie, of the form 1+2+3+...).

What are the lengths of the sides of the triangular and hexagonal tiles?

It is not necessary to calculate the chance of the disc landing on one tile. The following method uses the fact that this chance is the same for both tile shapes.

Hexagon:

Height (between opposite sides) H1 = √3 x side length = √3S1



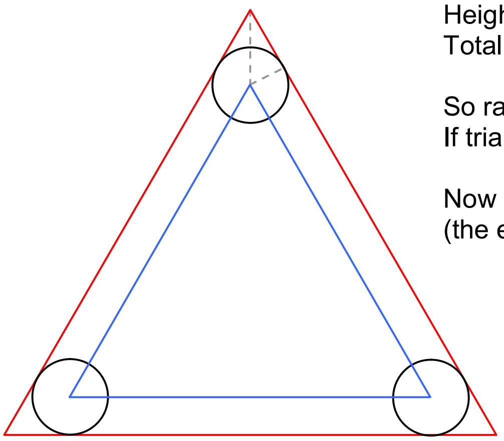
Random disc centre is somewhere within the red hexagon. To be completely within that one tile, the centre must be within the blue hexagon, which has it's height reduced by D.

Ratio of Blue/Red dimensions is (H1-D)/H1 = 1 – D/H1. Chance of disc falling completely on one tile (i.e. within blue) = ratio of areas, which is the square of this size ratio.

For triangle to have the same chance, it needs it's Blue/Red ratio (with the same disc) to be the same. So it must have a different height H2.

Triangle:

Red triangle has H2 = √3S2/2.   To fall within one tile the disc centre must fall within the blue triangle.
This example has the same sized disc as the pentagon.
Height reduction at top = D   (1/2 angle = 30º, sin 30 = 0.5, perp. = D/2).
Total height reduction (top + bottom) = 1.5D



So ratio of Blue/Red dimensions is (H2–1.5D)/H2 = 1 – 1.5D/H2
If triangle ratio = hexagon ratio then D/H1 = 1.5D/H2 or H2 = 1.5 x H1

Now H1 = √3S1   and   H2 = √3S2/2, so S2 = 3 x S1
(the example has been redrawn to this scale)

The list of even triangular numbers (see below) has one pair with dimensions <1000mm that are 3:1 apart, so this is the answer:

S1 (hexagon) = 210mm   S2 = (triangle) = 630mm.

6 10   28 36   66 78   120 136   190 210   276 300   378 406   496 528   630 666   780 820   946 990