Sunday Times Teaser 3040 – Moving Digit

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Jonny has opened a new bank account and has set up a telephone PIN. His sort code is comprised of the set of three two-figure numbers with the smallest sum which give his PIN as their product. He was surprised to find that the PIN was also the result of dividing his eight-figure account number by one of the three two-figure numbers in the sort code.

The PIN has an unusual feature which Jonny describes as a moving digit. If the number is divided by its first digit then the number which results has the same digits in the same order except that first digit is now at the end.

The account number does not contain the digit which moved.

What is the account number?

Solution by John Crabtree and Brian Gladman

Since the account number is eight digits and is the product of the PIN and a two-digit number, the minimum PIN is the minimum eight-digit number divided by the maximum two-digit number, which means that the PIN has 6 or 7 digits. But, since it is also the product of three two-digit numbers, it cannot have 7 digits. Therefore it has six digits and can be expressed in the form:

$$PIN = 10^5 d + x \tag{1}$$

where d is its first digit and x is a five-digit integer. The special form of the PIN results in the equation:

$$(10^5 d + x)/d = 10x + d \tag{2}$$

This can be easily solved for x as a function of d:

$$x = \frac{(10^5 - d)d}{10d - 1} \tag{3}$$

Equations (1) and (3) can now be combined to give the PIN as a function of d:

$$PIN = \frac{(10^3 - 1)(10^3 + 1)d^2}{10d - 1} = \frac{(3^3 \times 37) \times (7 \times 11 \times 13)d^2}{10d - 1} \tag{4}$$

This gives two solutions for first digits of 1 and 4:

$$PIN = \begin{cases} 111{,}111 & 3 \times 7 \times 11 \times 13 \times 37 \\ 410{,}256 & 2^4 \times 3^2 \times 7 \times 11 \times 37 \end{cases} \tag{5}$$

Since the PIN is the product of three two-digit integers with the minimum possible sum, we have to combine the prime factors in the PIN to give three two-digit divisors; and to minimise their sum we have to make their sizes as close to equal as possible. This means that they should each be close to $\sqrt[3]{410256} \sim 74$. This quickly leads to two possible solutions:

$$PIN = \begin{cases} 111{,}111 & (37, 39, 77) \\ 410{,}256 & (72, 74, 77) \end{cases} \tag{6}$$

The account number is now the product of the PIN and one of the three two-digit numbers in the sort code. The first possible solution cannot give an eight-digit account number while two of the three account numbers for the second — 29538432, 30358944 or 31589712 — are invalid because they include the 'moving' digit (4).

Hence the account number is 31589712 with the sort code $(72, 74, 77)$ and the PIN 410256.