Reservoir Development

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A straight track from an observation post, O, touches a circular reservoir at a boat yard, Y, and a straight road from O meets the reservoir at the nearest point, A, with OA then extended by a bridge across the reservoir's diameter to a disembarking point, B. Distances OY, OA and AB are whole numbers of metres, with the latter two distances being square numbers.

Following development, a larger circular reservoir is constructed on the other side of the track, again touching OY at Y, with the corresponding new road and bridge having all the same properties as before. For both reservoirs, the roads are shorter than 500m, and shorter than their associated bridges. The larger bridge is 3969m long.

What is the length of the smaller bridge?

Solution **Answer 2304m**

If radius of the larger circle is r, then Pythagoras gives OY^2 + r^2 = [OA + r]^2, so OY^2 = OA[OA + 2r], so OY^2 = OA.OB.

OA is a square, hence OB is a square and, since OB = OA + AB, √OA, √AB, √OB give a Pythagorean triple. For example: √OA=3, √AB=4, √OB=5, OY = √OA.√OB = 15

So √AB = √3969 = 63, OA + 63^2 = OB, with √OA = 1, 2, 3 ….22.
Testing gives the only whole number solution as √OA = 16, √OB = 65.
Hence OY = 1040m.

Correspondingly for the smaller circle we have 1040 = OA.[OA + AB].
Since AB is smaller in the smaller circle, √OA must be larger than 16 in this small circle.
For this smaller circle OA + AB = OB and √OA√OB = 1040.
So OA + AB = [1040^2]/OA, so AB = [1040^2]/OA – OA.
Testing √OA = 17, 18 ……..21, 22 gives √OA = 20, √AB = 48 as the only solution.
So original bridge AB = 48^2 = 2304m.