Sunday Times Teaser 3034 – Reservoir Development

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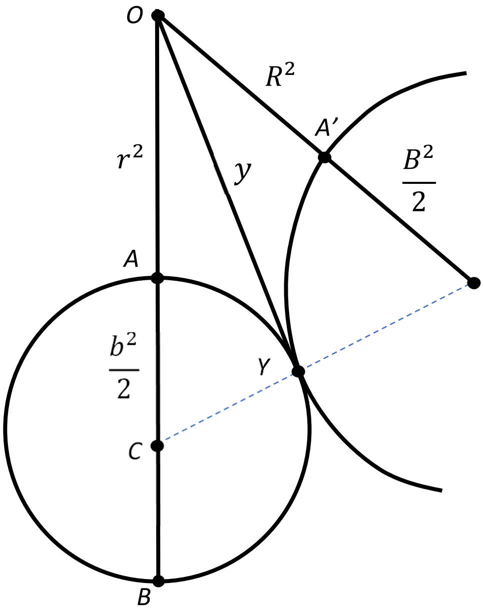

A straight track from an observation post, O, touches a circular reservoir at a boat yard, Y, and a straight road from O meets the reservoir at the nearest point, A, with OA then extended by a bridge across the reservoir's diameter to a disembarking point, B. Distances OY, OA and AB are whole numbers of metres, with the latter two distances being square numbers.

Following development, a larger circular reservoir is constructed on the other side of the track, again touching OY at Y, with the corresponding new road and bridge having all the same properties as before. For both reservoirs, the roads are shorter than 500m, and shorter than their associated bridges. The larger bridge is 3969m long.

What is the length of the smaller bridge?

Solution by Brian Gladman[1]

As shown on the diagram above, let the lengths of the roads from the observation post at *O* to the small and large reservoirs at A and A' be r^2 and R^2 respectively. Similarly let the lengths of the bridges be b^2 and B^2 and the length of the track from *O* to the boatyard at *Y* be y. Using the right-angled triangle *OYC:*

$$y^2 = (r^2 + b^2/2)^2 - (b^2/2)^2 = r^2(r^2 + b^2) \qquad (1)$$

For the larger reservoir we have:

$$B^2 + R^2 = (y/R)^2 \qquad (2)$$

Let the three sides of a right-angled triangle be B, R and $(R + x)$ so that:

$$B^2 + R^2 = (R + x)^2 = (y/R)^2 \qquad (3)$$

Noting that $x = \sqrt{R^2 + B^2} - R$ with $B^2 = 3969$ and $R^2 < 500$ (i.e. $B = 63$ and $0 < R < 23$), the range of x can be established as $45 \le x \le 63$.

Equation (3) can be used to express R in terms of B and x as:

$$R = (B^2/x - x)/2 \qquad (4)$$

which shows that x is a divisor of B^2 and, since this must be in the range $45 \le x \le 63$, the only candidate is $x = 49$ giving $R = 16$ and $y = R(R + x)$ as 1040 metres.

For the smaller reservoir:

$$b^2 = (y/r)^2 - r^2 \qquad (5)$$

where r is a factor of y. Since y is the same for both reservoirs, equation (1) shows that $b < B$ implies that $r > R$, which gives the range for r as $16 < r < 23$. In this case the only candidate is $r = 20$, giving $b = 48$ and hence [1]a smaller reservoir bridge length (b^2) of 2304 metres.

[1] *based on an analysis by John Crabtree.*