End of the Beginning

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Jenny is using her calculator, which accepts the input of numbers of up to ten digits in length, to prepare her lesson plan on large numbers. She can't understand why the results being shown are smaller than she expected until she realizes that she has entered a number incorrectly.

She has entered the number with its first digit being incorrectly entered as its last digit. The number has been entered with its second digit first, its third digit second etc. and what should have been the first digit entered last. The number she actually entered into her calculator was 25/43rds of what it should have been.

What is the correct number?

Solution **Answer: 530,864,197**

Let the correct number be "ab · · · · · ·" depending on the number of digits.

i) For two digits- ab = ba x 43/25 = 1.72ba
for "b" to be an integer, and as "a" can't be a 0, then "a" must be 5
so 5b = 1.72 x b5 : 50 + b = 1.72(10b + 5) = 17.2b + 8.6 : 16.2b = 41.4 and multiplying by 5 -
81b = 207 , as 81 doesn't divide exactly into 207, "b" can't be an integer so this can't be a solution.

ii) For three digits- abc = 1.72 bca , and again "a" must be 5.
So 500 + 10b + c = 1.72(100b + 10c + 5) : 162b + 16.2c = 491.4 multiplying by 5 -
810b + 81c = 2,457, again 81 doesn't divide exactly into 2,457 so three digits doesn't have a solution.

iii) For four digits- 8,100b + 810c + 81d = 24,957 - no solution

iv) For five digits- 81,000b + 8,100c + 810d + 81e = 249,957 - no solution

A pattern is emerging as shown in the following table-

Number of Digits	**RHS Value**
2	207
3	2,457
4	24,957
5	249,957
6	2,499,957
7	24,999,957
8	249,999,957
9	2,499,999,957
10	24,999,999,957

The number of digits in the RHS value being tested for divisibility by 81 is too many for most calculators so a short cut non-calculator test is applied in the following table. To test for divisibility by 81 the last digit of the number is removed and from the remaining number, 8 times the removed last digit is deducted. If after repeated application exactly 0 is left then the number is divisible by 81.

Number of Digits								
2	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**
207	2,457	24,957	249,957	2,499,957	24,999,957	249,999,957	2,499,999,957	24,999,999,957
-36	189	2,439	24,939	249,939	2,499,939	24,999,939	249,999,939	2,499,999,939
	-54	171	2,421	24,921	249,921	2,499,921	24,999,921	249,999,921
		9	234	2,484	24,984	249,984	2,499,984	24,999,984
			-9	216	2,466	24,966	249,966	2,499,966
				-27	198	2,448	24,948	249,948
					-45	180	2,430	24,930
						18	243	2,493
						-63	**0**	225
								-18

Only for 9 digits is the RHS value exactly divisible by 81 giving a quotient of 30,864,197.

So $10^7b + 10^6c + 10^5d + 10^4e + 10^3f + 10^2g + 10h + i = 30,864,197$

As the first digit of the correct number, "a" is 5, the correct number is 530,864,197.