

Party Time

Graham Smithers

A four-digit number with different positive digits and with the number represented by its last two digits a multiple of the number represented by its first two digits, is called a PAR.

A pair of PARs is a PARTY if no digit is repeated and each PAR is a multiple of the missing positive digit.

I wrote down a PAR and challenged Sam to use it to make a PARTY. He was successful.

I then challenged Beth to use my PAR and the digits in Sam's PAR to make a different PARTY. She too was successful.

What was my PAR?

Solution**Answer: 1785**

Possible PARs [Increasing order]	Possible larger PARs
1236	-
1248	-
1284	-
1296	-
1326	-
1352	4896
1365	2987
1378	2496, 4692
1428	-
1456	2987, 3978✓
1498	3672
1632	-
1648	-
1734	2856, 2958✓
1768	-
1785	2346, 2369, 2496✓, 3264, 3296, 4692✓
1836	2754✓
1854	2369, 3296, 3672✓
1872	-
1938	2754✓
1957	2346, 3264, 3468, 4386

None of the remaining possibles 1976, 2163, 2184, 2346, 2369, 2496, 2678, 2754, 2781, 2856, 2958, 2987, 3162, 3264, 3296, 3468, 3672, 3876, 3978, 4182, 4386, 4692, 4896 have associated possible larger PARs. Of the above possibilities, those with a ✓ give possible PARTYs:

- 1456 and 3978, missing digit 2 a common factor
- 1734 and 2958, missing digit 6 a common factor
- 1785 and 2496, missing digit 3 a common factor ✓✓
- 1785 and 4692, missing digit 3 a common factor ✓✓
- 1836 and 2754, missing digit 9 a common factor
- 1854 and 3672, missing digit 9 a common factor
- 1938 and 2754, missing digit 6 a common factor

So my PAR was 1785 and Sam's and Beth's PARs were 2496 and 4692 in either order.