

Pot Success

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In snooker, pot success (PS) is the percentage of how many pot attempts have been successful in that match (e.g. 19 pots from 40 attempts gives a PS of 47.5). In a recent match, my PS was a whole number at one point. I then potted several balls in a row to finish a frame, after which my improved PS was still a whole number. At the beginning of the next frame, I potted the same number of balls in a row, and my PS was still a whole number. I missed the next pot, my last shot in the match, and, remarkably, my PS was still a whole number.

If I told you how many balls I potted during the match, you would be able to work out those various whole-number PS values.

How many balls did I pot?

Solution**Answer: 13**

At the point in the match where my PS is a whole number, let a be the number of pots and b be the number of attempts up to that point, so $100a/b$ is a whole number (I assume that a cannot be zero). After then potting n balls, my PS has increased to $100(a+n)/(b+n)$, and after a further n pots it is $100(a+2n)/(b+2n)$. After the final miss, the PS is $100(a+2n)/(b+2n+1)$. Thus, all of the following numbers are whole:

$$100a/b, 100(a+n)/(b+n), 100(a+2n)/(b+2n) \text{ and } 100(a+2n)/(b+2n+1)$$

Now let $x = 100(a+2n)/(b+2n)$ and $y = 100(a+2n)/(b+2n+1)$, so $x/y = (b+2n+1)/(b+2n)$, or $y = (x-y)(b+2n)$

I potted "several" balls in a row, so n must be at least, say, 3, and $b+2n+1$ is at least 8.

Then we have $x/y = 8/7, 9/8, 10/9, \dots$ etc. For the first of these, we could have $(x,y) = (8,7), (18,14), \dots (96, 84)$. x and y are PS values so can't be more than 100, so $x-y$ is no more than 12. Subsequent values of x/y will not allow $x-y$ to be any more than 12. Try $x-y = 1, 2, 3 \dots$ in turn:

$x-y = 1$, so $y = b+2n$

$100(a+2n) = x(b+2n) = y(y+1)$, so $y(y+1)/100$ is whole. The only possibilities for y are 24 and 75.

If $y=24$, $a+2n=6$, but n is 3 or more, so that doesn't work.

If $y=75$, $a+2n=57$ and $b+2n=75$, so $b-a=18$, both a and b are odd, and $100a/b$ is whole, so b is divisible by 5. The only possibilities that work are $(a,b)=(7,25)$ and $(27,45)$.

$x-y = 2$, so $y = 2(b+2n)$ and $a+2n = y(y+2)/200$ which must be whole. The only possibilities for y are 48 and 50.

If $y=48$, $a+2n=12$ and $b+2n=24$, so $b-a=12$, both a and b are even and $100a/b$ is whole. The only possibility is $(a,b)=(4,16)$.

If $y=50$, $a+2n=13$ and $b+2n=25$, so $b-a=12$, both a and b are odd and $100a/b$ is whole. The only possibility is $(a,b)=(3,15)$.

$x-y = 3$, so $y = 3(b+2n)$ and $y(y+3)/300$ is whole. The only possibility is $y=72$ and $(a,b)=(0,6)$ - not allowed.

$x-y = 4, 6, 7, 8, 9, 11$ and 12 give no solutions. For example, if $x-y=7$, $y=7(b+2n)$ and $y(y+7)/700$ is whole, so y is a multiple of 7 and y or $y+7$ must be a multiple of 25, which doesn't work.

$x-y = 5$, so $y = 5(b+2n)$ and $y(y+5)/500$ is whole. The only possibility is $y=75$.

Then $a+2n=12$ and $b+2n=15$, b is odd and $100a/b$ is whole. The only possibility is $(a,b)=(2,5)$

$x-y = 10$, so $y(y+10)/1000$ is whole. The only possibility is $y=50$, but $a+2n=3$, which is impossible

Summarising the various possibilities with the PS values at the various stages:

a	b	n	PS	PS	PS	PS	Balls potted	Balls attempted
7	25	25	28	64	76	75	57	76
27	45	15	60	70	76	75	57	76
4	16	4	25	40	50	48	12	25
3	15	5	20	40	52	50	13	26
2	5	5	40	70	80	75	12	16

Only if I potted 13 balls can you work out all the PS values.