Square jigsaws

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I chose a whole number and asked my grandson to cut out all possible rectangles with sides a whole number of centimetres whose area, in square centimetres, did not exceed my number. (So, for example, had my number been 6 he would have cut out rectangles of sizes 1×1, 1×2, 1×3, 1×4, 1×5, 1×6, 2×2 and 2×3.) The total area of all the pieces was a three-figure number of square centimetres.

He then used all the pieces to make, in jigsaw fashion, a set of squares. There were more than two squares and at least two pieces in each square.

What number did I originally choose?

Solution to 'Square jigsaws' **Answer: 29**

If my number is N, then one of the square jigsaws is at least N×N (to accommodate the 1×N piece). Also, to use at least two pieces, a 1×1 and 2×2 jigsaw are impossible. Furthermore, we can soon see that there are not enough small pieces to make two separate 3×3 jigsaws. Therefore the total minimum area of the rectangles must be at least $N^2 + 9 + 16$.

If N=15 a quick count gives a total area of the pieces as 210, way short of $15^2 + 25$. For subsequent N we calculate the areas cumulatively below:

Number N	**Rectangles of area =N**	**Area of those rectangles**	**Total area T of all rectangles**	**$T-N^2 \geq 25$?**
16	3	48	258	
17	1	17	275	
18	3	54	329	
19	1	19	348	
20	3	60	408	
21	2	42	450	
22	2	44	494	
23	1	23	517	
24	4	96	613	37
25	2	50	663	38
26	2	52	715	39
27	2	54	769	40
28	3	84	853	69
29	1	29	882	41
30	4	120	>999	

In no case is $T \geq (N+1)^2 + 25$ and so the jigsaws are N×N and at least two others totalling $T - N^2$ in area. Of those numbers listed in the right-hand column above, only 41 can be expressed as a sum of some of 9, 16s, 25s, 36, 49 and 64. So the only possibility is N=29 and all the pieces in this case *can* be used to make square jigsaws of sides 29, 5 and 4.

[For completeness, one such possible layout can be seen on the next sheet.]