Rainbow Numeration

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Dai had seven standard dice, one in each colour of the rainbow (ROYGBIV). Throwing them simultaneously, flukily, each possible score (1 to 6) showed uppermost. Lining up the dice three ways, Dai made three different seven-digit numbers: the smallest possible, the largest possible, and the 'rainbow' (ROYGBIV) value. He noticed that, comparing any two numbers, only the central digit was the same and each number had just one prime factor under 10 (different for each number).

Hiding the dice from his sister Di's view, he told her what he'd done and noticed, but wanted her to guess the 'rainbow' number digits in ROYGBIV order. Luckily guessing the red and orange dice scores correctly, she then calculated the others unambiguously.

What score was on the indigo die?

Solution to 'Rainbow numeration' **Answer: 4**

With each possible score - 1, 2, 3, 4, 5 and 6 uppermost the 7th score must repeat one of these.
Each 7-digit number has just one single-figure prime factor - different for each. So the repeated score can't be 3 or 6 - otherwise digit sum div. by 3 and so for each permed number.

So initial possibilities are:-

SMALLEST	LARGEST	ROYGBIV
1123456 [factor 2 - not 3, 5 or 7]	6543211 [not factors 2,3,5 or 7]	n/a
1223456 [factor 2 - not 3, 5 or 7]	6543221 [not factors 2,3,5 or 7]	n/a
1234456 [factor 2 - not 3, 5 or 7]	6544321 [factor 7- not 2, 3 or 5]	see below
1234556 [factor 2 - not 3, 5 or 7]	6554321 [not factors 2,3,5 or 7]	n/a

Central 4 in 1234456=2xN and 6544321=7xM also in ROYGBIV (so G=4). ROY4BIV must also be div. by 5, so V=5 and rainbow value is ROY4BI5 with R, O, Y, B and I not in same digit positions as 1234456 and 6544321.

[R][O][Y]4[B][I]5 perms from [2,3,4][1,3,4,6][1,2,6]4[1,2,6][1,3,4,6]5 with clear restrictions.

2164??5 and 2614??5 - invalid - impossible to perm from above

2314645=5x.. - valid, but 2364145=5x7x.. - invalid - unambiguous given R and O

24[1,6]4[1,6]35 - ambiguous perms - 2414635=5x.. and 2464135=5x.. - no factor 7

3124645=5x.. - valid, but 3164245=5x7x.. - invalid - unambiguous given R and O

34[1,2,6]4[1,2,6][1,6]5 - ambiguous perms - 3414265=5x.. and 3424165=5x.. - no factor 7
- 3424615=5x.. and 3464215=5x.. - no factor 7

3614245=5x.. - valid, but 3624145=5x7x.. - invalid - unambiguous given R and O

41[2,6]4[2,6]35 - ambiguous perms - 4124635=5x.. and 4164235=5x.. - no factor 7

43[1,2,6]4[1,2,6][1,6]5 - ambiguous perms - 4314265=5x.. and 4324165=5x.. - no factor 7
- 4324615=5x.. and 4364215=5x.. - no factor 7

46[1,2]4[1,2]35 - ambiguous perms - 4614235=5x.. and 4624135=5x.. - no factor 7

For each unambiguous case above, I(ndigo die)=4