

Long Shot

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Callum and Liam play a simple dice game together using standard dice (numbered 1 to 6). A first round merely determines how many dice (up to a maximum of three) each player can use in the second round. The winner is the player with the highest total on their dice in the second round.

In a recent game Callum was able to throw more dice than Liam in the second round but his total still gave Liam a chance to win. If Liam had been able to throw a different number of dice (no more than three), his chance of winning would be a whole number of times greater.

What was Callum's score in the final round?

Solution**Answer 10**

In the table below the outcome frequency of possible scores is given for 1, 2 and 3 dice (e.g. there are 21 ways of obtaining a score of 13 with 3 dice from a possible 216 outcomes).

This is followed by the frequency of beating that score with that number of dice (e.g. the number of ways of beating 13 with three dice is the sum of the ways of getting 14, 15, 16, 17 and 18, i.e. $15+10+6+3+1 = 35$).

Score	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1 die	1	1	1	1	1	1												
	5	4	3	2	1	0												
2 dice		1	2	3	4	5	6	5	4	3	2	1						
	36	35	33	30	26	21	15	10	6	3	1	0						
3 dice			1	3	6	10	15	21	25	27	27	25	21	15	10	6	3	1
	216	216	215	212	206	196	186	160	135	108	81	56	35	20	10	4	1	0

The probability of Liam winning is given by the frequency in the table divided by 6, 36 or 216 as appropriate (e.g. if Callum scores 13, Liam's chances with three dice are $35/216$).

Only in the case of a score of **10** by Callum with three dice are the conditions of the puzzle met.

Liam can beat him with two dice (with two 6s or a 5 and a 6), and if he can use three dice, his chances increase from 3 in 36 to 108 in 216, i.e. from 1 in 12 to 6 in 12, so a whole number of times greater.